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A sustainability transition on the move? Evidence based on the disconnect from market fundamentals

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Executive summary

Over recent years, to foster a “green transition” of the economy, various European Union (EU) policies were implemented to shift financial investors’ preferences towards sustainable investments. Against this background, a concern from a financial stability angle is whether a strong push towards sustainable investments might increase the risk of creating asset price bubbles in relevant market niches, which are perceived as more sustainable from an environmental, social and governance (ESG) perspective. At the same time, in the context of a low-carbon transition, high-carbon assets could become stranded and associated investments could depreciate, posing risks to the stability of the financial system. These arguments become even more relevant in view of stock market developments over the past two years, with European equities hitting an unprecedented record high even when compared with 2001 and 2007 levels, with the real economy only gradually recovering from the Covid-19 pandemic.

To shed some light on these issues, we investigate to what extent stock market prices are disconnected from their ‘fundamentals’, defined as past prices and dividends. To do so we use monthly data on the European stock market from 2005 to 2022, analysing the European market as a whole, as well as the green, high-carbon and ESG segments.

We find evidence that at the beginning of 2022 the non-fundamental component in the European stock market was about 25% of the total price, a record-high never observed before. When looking at particular portfolios, the model shows that green and ESG stocks behave broadly in line with the market. However, in recent years ESG stocks have shown a significant, though small, overvaluation compared to the market. In addition, the model shows that markets are uncertain about the actual value of high-carbon assets, as reflected in the huge uncertainty surrounding the estimates of the non-fundamental component for these particular stocks.

We interpret this finding in the context of an evolving EU sustainable finance regulation as a successful shift of investors’ preferences towards sustainability, suggesting a “transition on the move” in financial markets. Moreover, while the disconnect of the ESG segment deserves careful monitoring, the major concern to policymakers should probably be the substantial overvaluation in the entire market.

A sustainability transition on the move? Evidence based on the disconnect from market fundamentals*

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Abstract

In a context where European stock prices have been trending upwards, one of the main concerns is that stocks perceived as more sustainable from an environmental, social and governance (ESG) perspective could be particularly exposed to exuberance. To shed some light on the magnitude of the deviation of stock prices from fundamentals we apply a Markov-switching augmented version of the present-value model. Using monthly data on the European stock market from 2005 to 2022, our model suggests that at the beginning of 2022 the non-fundamental component was about one fourth of the total price. When looking at particular market segments, the model shows that green and ESG stocks behave broadly in line with the market. However, in recent years ESG stocks have shown a significant, though small, disconnect from the market. These findings suggest that investor preferences are shifting towards sustainability, while not posing immediate risks to market stability.

JEL Classification: C11, C32, G12.

Keywords: Bayesian inference, European stock market, green transition, Markov-switching, present-value model.

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1 Introduction

Over recent years, to foster a “green transition” of the economy various European Union (EU) policies were implemented to shift investors’ preferences towards sustainable investments. In this context, a concern from a financial stability perspective is whether a strong push towards sustainable investments might increase the risk of creating asset price bubbles in relevant market niches. At the same time, in the the context of a low-carbon transition, high-carbon assets could become stranded and associated investments could depreciate, posing risks to the stability of the financial system. These arguments become even more relevant in view of stock market developments over the past two years, with European equities hitting an unprecedented record high even when compared with 2001 and 2007 levels, with the real economy gradually recovering from the consequences of the Covid-19 pandemic.

Already after the great financial crisis, a debate started whether exceptionally easy monetary conditions could create pressures on asset prices to deviate from their fundamental values, i.e. the factors underlying the actual financial performance of a company (e.g. see [BIS, 2015](#); [Blanchard and Gagnon, 2016](#)). In Europe, the European Central Bank (ECB) established several Quantitative Easing (QE) programmes, thereby directly purchasing financial assets. By today’s perspective there is evidence that periods of QE coincided with exuberant investor behaviour, reflected in a moderate disconnect of equities from their fundamentals, even when controlling for improved macro fundamentals (see e.g. [Hudepohl et al., 2021](#)).

Looking at stocks with a particularly good environmental, social and governance (ESG) performance, general market conditions have combined with the evolving EU sustainable finance regulation aimed at shifting investors’ preferences to hold sustainable assets.¹ For this reason, developments in this market segment are monitored carefully by investors and regulators, as after all, green tech or alternative energy bubbles have indeed taken place

¹In 2018, the European Commission (EC) launched an Action Plan on financing sustainable growth, followed by a renewed strategy in 2021. The main objectives are to reorient capital flows towards a more sustainable economy and mainstreaming sustainability into risk management while fostering transparency and long-termism. <https://ec.europa.eu/info/publications/sustainable-finance-renewed-strategy-en>

since the mid-2000s. Back then, as discussed in [Bohl et al. \(2013, 2015\)](#), Europe and in particular Germany faced a boom and bust in alternative energy stocks driven by investors' sentiment. In response to the intense sector competition and the downturn due to the global financial crisis, profit margins eventually declined as well as stock prices. In fact, the underlying dynamics could not be explained by rising oil prices or a general stock market euphoria.

On the back of this experience, the question today is whether we might be facing a similar situation of particular assets being potentially inflated beyond the sector's growth potential. Hence, by early 2021, concerns were expressed on the formation of a green bubble², which by the second half of 2021 extended to the ESG sphere as a whole and entered the policy discussion.³ In addition, evidence was brought forward by a counterfactual study (see [van der Beek, 2021](#)) on the US that argued that the aggregate ESG industry would have strongly underperformed the market from 2016 to 2021, should the unexpected inflows from institutional funds not have taken place.⁴

Despite past examples of asset price bubbles, deviations from stock market fundamentals do not necessarily point to unsustainable excesses, as they are often on productive assets in an expanding sector of the economy. An inflated market or sector might also produce positive spillovers to other sectors, when it alleviates unjustified financing constraints (see [Anderson et al., 2010](#); [Campello and Graham, 2013](#)). In general, a deviation from fundamentals could well be justified by rational expectations on economic developments rooted e.g. in

²19 February 2021: "Green bubble' warnings grow as money pours into renewable stocks" <https://www.ft.com/content/0a3d0af8-7092-44c3-9c98-a513a22629be>; 20 May 2021: "A green bubble? We dissect the investment boom" <https://www.economist.com/finance-and-economics/2021/05/17/green-assets-are-on-a-wild-ride>; 24 May 2021: "Clean energy stocks are as crowded as tech before dotcom crash, says MSCI" <https://www.ft.com/content/74baff9a-bce6-49a5-b7f5-7cbf84ac32c6>.

³See [BIS \(2021\)](#) "There are signs that ESG assets' valuations may be stretched [...] Even after a decline from their peak in January 2021, price-to-earnings ratios for clean energy companies are still well above those of already richly valued growth stocks". 22 October 2021: "ESG will create bubbles and the next Amazon or Tesla" <https://www.cnbc.com/2021/10/22/esg-will-create-bubbles-and-the-next-amazon-or-tesla-iif.html>. 26 October 2021: Analysis on price-earnings ratios by Banque de France (see [Jourde and Stalla-Bourdillon, 2021](#)) on green stocks contends that firms with a high environmental score appear to have lower valuations than their high-carbon counterparts. 28 October 2021: "Trillion-Dollar ESG Boom Rings Bubble-Trouble Alarm in New Study" <https://www.bloomberg.com/news/articles/2021-10-28/trillion-dollar-esg-boom-rings-bubble-trouble-alarm-in-new-study>.

⁴[Griffin et al. \(2011\)](#) makes a similar argument with reference to the tech bubble in the US. At the end of the 1990s, technology stocks quintupled due to a shift in demand by institutional investors (primarily hedge funds, independent investment advisors and mutual funds) to the technology sector. Later, the broad sell-off of institutional investors and households triggered the collapse.

policy objectives or other information that is available to investors. The growth of rational bubbles is consistent with rational expectations and reflects the presence of expectations about future increases in an asset's price.⁵

Because of the forward-looking nature of investment decisions and the large information set they are based on, no econometric model - including ours - would be able to tell whether an identified disconnection between stock prices and their fundamentals in a particular market segment is driven by rational behaviour or irrational exuberance. An alternative strand of literature uses behavioural models to allow for irrational pricing and the appearance of "irrational" bubbles (see [Vissing-Jorgensen, 2004](#)).

The empirical literature identifying abnormal market developments follows primarily two prominent strands that have grown rapidly since the great financial crisis.⁶ The first group applies cointegration and unit root tests in the context of an indirect bubble test looking for explosive behaviour (see [Diba and Grossman, 1988a](#); [Phillips et al., 2011, 2015](#); [Homm and Breitung, 2012](#)). The other group applies a Markov-switching-augmented version of the present-value model (see [Van Norden and Schaller, 1999](#); [Brooks and Katsaris, 2005](#); [Binsbergen and Kojen, 2010](#); [Al-Anaswah and Wilfing, 2011](#); [Choi et al., 2017](#); [Chan and Santi, 2021](#)).⁷ While each of these approaches has its advantages and shortcomings, we follow the latter approach as it provides the advantage of being able to unveil a non-fundamental component and the related estimation uncertainty. In particular, we employ a model that builds on latest developments in the literature on present-value models. In this framework, market fundamentals are a function of expected dividend growth and returns, while the non-fundamental component is assumed to follow a Markov-switching process that allows for the possibility of exploding and collapsing regimes. Although our model is not capable to explicitly incorporate additional information related to the evolving state of macroeco-

⁵For an exhaustive discussion on the theory of "rational" bubbles in the context of asset pricing see [Diba and Grossman \(1988b\)](#); [Kortian \(1995\)](#); [Santos and Woodford \(1997\)](#), in the context of systemic risk [Brunnermeier and Oehmke \(2013\)](#), and in the context of broader macroeconomics [Martin and Ventura \(2018\)](#). Differences among "rational" and "irrational" bubbles are discussed in [Meltzer \(2002\)](#), [Shi and Suen \(2014\)](#) and [Balcombe and Fraser \(2017\)](#).

⁶For an exhaustive overview see [Gürkaynak \(2008\)](#) and [Chan and Santi \(2021\)](#).

⁷Besides these two groups, two other approaches are proposed by the literature to assess the fair value of assets. [Cecchetti and Taboga \(2017\)](#) use a probabilistic framework that allows to simultaneously take into account asset prices and economic determinants, while [Binswanger \(2004\)](#) and [Velinov and Chen \(2014\)](#) use a Markov-switching structural vector autoregressive (MS-SVAR) model to detect fundamentals.

conomic determinants (Cecchetti and Taboga, 2017), market power considerations (Farhi and Gourio, 2019), issues regarding the flow of funds (van der Beck, 2021) or the structural decline in the natural rate of interest (Monache et al., 2021), it provides a parsimonious representation of market dynamics which is in line with the business cycle. Finally, for the estimation of the model we propose an alternative Bayesian approach to those available in the literature, which is more efficient.

Our empirical study of the alignment between equity prices and their fundamentals in various relevant market niches over time, is based on a sample including 1200 stocks traded in the main European markets from January 2005 to January 2022. Our findings suggest that the non-fundamental component in the EU stock market at the beginning of 2022 accounts for about twenty-five per cent of the total price. Based on a comparison of density estimates of non-fundamental components, we do not find evidence of much higher overvaluation in the green or in the broader sustainability segment. Indeed, the weight of the non-fundamental component for the green portfolio is in line with that of the market. With respect to the ESG portfolio it appears to be slightly more overvalued than the market as a whole, which however has been the case for some years already. We interpret these results as EU legislation being successful in shifting investor preferences and hence flows of funds towards sustainability objectives, suggesting a “transition on the move” in financial markets. In addition, the model shows that markets are uncertain about the actual value of high-carbon assets, as reflected in the huge uncertainty surrounding the estimates of the non-fundamental component for these particular stocks. Overall, the ESG segment does not seem to be much more inflated than the market, but the market is significantly overvalued, which can be a source of financial risk.

The remainder of this paper is structured as follows. Section 2 outlines the model. Section 3 describes the data. Section 4 illustrates the results for the EU market as well as the green, high-carbon and ESG portfolios, and contrasts the non-fundamental components across portfolios. Section 5 concludes.

2 The present-value model

In the tradition of [Campbell and Shiller \(1988\)](#), we start from a log-linearized version of the present-value model. Following for instance [Binsbergen and Kojen \(2010\)](#) and [Choi et al. \(2017\)](#), we define pd_t as the logarithm of the price-dividend ratio, $pd_t = \ln(P_t/D_t)$, and Δd_t as the logarithm of the dividend growth rate, $\Delta d_t = \ln(D_t/D_{t+1})$. By resorting to a first order log-linear approximation, the log gross return, $r_t = \ln((P_t + D_t)/P_{t-1})$, can be specified as a linear function of the price-dividend ratio and the dividend growth rate as in:

$$r_{t+1} \simeq \kappa + \rho pd_{t+1} + \Delta d_{t+1} - pd_t \quad (1)$$

with $\kappa = \ln(1 + \exp(\bar{pd})) - \rho \bar{pd}$, $\rho = \exp(\bar{pd}) / (1 + \exp(\bar{pd}))$, and $\bar{pd} = (1/T) \sum_{t=1}^T pd_t$. Iterating forward Equation (1) and taking the expectation conditional on today's information, $\mathfrak{S}_t = (\Delta d_1, \Delta d_2, \dots, \Delta d_t, pd_1, pd_2, \dots, pd_t)$, yields the price-dividend fundamental component:

$$pd_t^f = \frac{\kappa}{1 - \rho} + \sum_{j=1}^{\infty} \rho^{j-1} E[\Delta d_{t+j} - r_{t+j} | \mathfrak{S}_t] \quad (2)$$

In line with [Binsbergen and Kojen \(2010\)](#) and [Chan and Santi \(2021\)](#), we assume the growth rate of dividends and gross returns equal their expected values plus an orthogonal shock as in:

$$\begin{aligned} d_t &= g_{t-1} + \epsilon_t^d \\ r_t &= \mu_{t-1} + \epsilon_t^r \end{aligned} \quad (3)$$

The latent expected gross returns, $\mu_t \equiv E[r_{t+1} | \mathfrak{S}_t]$, and expected dividend growth rate, $g_t \equiv E[\Delta d_{t+1} | \mathfrak{S}_t]$ follow the autoregressive processes:

$$\begin{aligned} \mu_t &= \alpha^\mu + \phi^\mu (\mu_{t-1} - \alpha^\mu) + \epsilon_t^\mu \\ g_t &= \alpha^g + \phi^g (g_{t-1} - \alpha^g) + \epsilon_t^g \end{aligned} \quad (4)$$

where α^μ and α^g are the unconditional mean of the expected gross returns and dividend respectively. Given (3) and (4), taking the conditional expectation in (1) allows to unveil a closed form solution for the fundamental price-dividend ratio:

$$pd_t^f = \frac{\kappa - \alpha^\mu + \alpha^g}{1 - \rho} - \frac{\mu_t - \alpha^\mu}{1 - \rho\phi^\mu} + \frac{g_t - \alpha^g}{1 - \rho\phi^g} \quad (5)$$

When the transversality condition does not hold the particular solution given in (5) is complemented by a non-fundamental or rational bubble component, say b_t . In this case the price-dividend ratio is expressed as the sum of a fundamental and a non-fundamental component as in:

$$pd_t = pd_t^f + b_t \quad (6)$$

In the equation above b_t satisfies the homogeneous difference equation $E[b_{t+i}|\mathfrak{S}_t] = \frac{b_t}{\rho^i}$. In line with other studies (e.g. [Al-Anaswah and Wilfling, 2011](#)), we assume that the non-fundamental component follows the autoregressive process:

$$b_t = \frac{b_{t-1}}{\rho} + \epsilon_t^b \quad (7)$$

With the aim of identifying periodically collapsing bubbles, we superimpose a two-regime Markov switching process, say S_t on equations (3)-(7). The econometric model is described by the following observational equations:

$$\begin{aligned} d_t &= g_{t-1} + e_{S_t}^d \\ pd_t &= pd_t^f + b_t \end{aligned} \quad (8)$$

The dynamics of the latent states is given by:

$$\begin{aligned} pd_t^f &= \alpha_{S_t}^p - (\mu_t - \alpha_{S_t}^\mu)/(1 - \rho_{S_t}\phi_{S_t}^\mu) + (g_t - \alpha_{S_t}^g)/(1 - \rho_{S_t}\phi_{S_t}^g) \\ b_t &= b_{t-1}/\rho_{S_t} + e_{S_t}^b \\ g_t &= \alpha_{S_t}^g + \phi_{gS_t}(g_{t-1} - \alpha_{S_t}^g) + e_{S_t}^g \\ \mu_t &= \alpha_{S_t}^\mu + \phi_{\mu S_t}^\mu(\mu_{t-1} - \alpha_{S_t}^\mu) + e_{S_t}^\mu \end{aligned} \quad (9)$$

Where $\alpha_{S_t}^p = (\ln(1 + \exp(\bar{p}d)) - \rho_{S_t}\bar{p}d - \alpha_{S_t}^\mu + \alpha_{S_t}^g)/(1 - \rho_{S_t})$. The variable S_t take values in $\{0, 1\}$, and its dynamic is ruled by the transition probabilities $\pi_{ij} = \Pr(S_t = i | S_{t-1} = j)$, $i, j = 0, 1$, and $t = 1, 2, \dots, T$. The 0-state identifies surviving regimes, i.e. $1/\rho_0 > 1$, while collapsing regimes prevail when $S_t = 1$ and $1/\rho_1 < 1$. The vector of shocks, $\epsilon_{S_t} = (e_{S_t}^d, e_{S_t}^b, e_{S_t}^g, e_{S_t}^\mu)$, is a function of the unit variance white noise vector $e_t = (e_t^d, e_t^b, e_t^g, e_t^\mu)$, i.e. $\epsilon_{S_t} = L_{S_t}e_t$, where $C_{S_t} = L_{S_t}L_{S_t}'$ is a Markov dependent covariance matrix. To simplify the model structure we set to zero the off-diagonal elements of C_{S_t} , while its diagonal entries take the form:

$$C_{S_t}(y, y) = \begin{cases} V^y & \text{if } S_t = 0 \\ V^y\delta^y & \text{if } S_t = 1 \end{cases} \quad (10)$$

where $y = d, b, g, \mu$, and $\delta^y > 1$ imposes shocks with larger variances during collapsing regimes. Appendix B details the Bayesian approach used to estimate the model described by equations (8)-(10): (i) the prior distribution elicited on model parameters $\theta = (\rho_j, \alpha_j^x, \phi_j^x, V^y, \delta^y)$, $j = 1, 2$, $x = g, \mu$, $y = d, b, g, \mu$, and on the transition probabilities of the Markov process π_{ij} , $j = 1, 2$; (ii) the MCMC algorithm employed to produce draws from the posterior distribution of both model parameters and the latent state $\xi_t = (pd_t^f, b_t, g_t, \mu_t)$.

3 Data description

We use monthly data from Refinitiv Datastream for 1200 European companies from January 2005 to January 2022 to build market, ESG, green and high-carbon portfolio data. The model requires two observable variables, the price-dividend ratio and dividend growth. Both require to be created applying different procedures as described below⁸. We obtain the aggregate price index of a portfolio by summing over the price index of individual firms, which is the product of the unadjusted price and number of shares. We obtain the aggregate dividend yield by weighting each firm's dividend yield with its market value. The price-dividend ratio is given by taking the natural logarithm of the aggregate price index divided

⁸The procedure is able to recover the aggregate Datastream portfolio data for the United States from the underlying constituents data.

by the aggregate dividend yield. The second observable is given by the natural logarithm of the rate of change in the aggregate dividend yield. The starting date varies across the different portfolios due to data issues at the beginning of some samples.⁹

TABLE 1 Time-varying amount of companies across portfolios

	Market	ESG	Green	High-carbon
2005	678	71	26	474
2006	718	84	55	447
2007	777	124	90	435
2008	825	162	102	453
2009	837	175	104	456
2010	846	189	110	451
2011	859	206	115	453
2012	878	215	122	468
2013	891	221	129	468
2014	920	232	139	469
2015	961	249	151	488
2016	1016	267	154	536
2017	1048	306	157	557
2018	1100	364	162	605
2019	1154	405	162	657
2020	1193	447	162	696
2021	1191	431	162	694
2022	1192	431	162	695

Notes: The table displays the time-varying dummies for green and high-carbon portfolios as provided by Alessi et al. (2021). The sample is only sparsely filled in the early years and we extrapolate dummies from 2019 to 2020 and 2021. Similarly we fill missing ESG data in 2021 with 2019 and extrapolate 2021 to 2022.

Table 1 displays the number of companies in each portfolio at each point in time. All portfolios are characterised by a time-varying composition, as the firms satisfying the definition may vary from a year to another. For the market or composite portfolio, we include all companies available in the dataset. For the ESG portfolio, we select all companies with a Datastream ESG score higher than 50.¹⁰ Environmental portfolios for green and high-

⁹A more detailed description of the formulas can be found in Appendix ???. The list of companies and the associated mnemonic codes are available upon request.

¹⁰The ESG score varies from 0 to 100. A higher threshold would result in too few companies. Whenever

carbon companies are built based on the methodology proposed in [Alessi et al. \(2021\)](#). In particular, the green portfolio includes firms characterised by lower emission intensity and higher transparency on their environmental performance, while the high-carbon portfolio includes firms that do not disclose on their environmental performance, and in addition, are active in fossil-fuel or energy-intensive sectors. It is noteworthy that in more recent years about one-third of the companies included in the ESG portfolio constitute about ninety per cent of the companies in the green portfolio. A plot of the raw data is provided in Appendix A.

The model presented in section 2 implies a departure from joint normality of the unconditional distribution of dividend growth rate and price dividend ratio. To check whether the data show evidence of asymmetry and fat tails, we focus on the following third and fourth-order moments:

$$m_{ij} = E[\tilde{\Delta d}_t^i \cdot \widetilde{\Delta pd}_t^j]$$

where \tilde{x} refers to the centered and standardised version of variable x , and $i, j = 0, 1, 2, 3, 4$, with $i+j = 3$ or $i+j = 4$. Assuming linearity and Gaussian errors, the third moments above are null, while the fourth-order moments are: equal to 3, when $i = 0, j = 4$ or $i = 4, j = 0$, equal to $3 * \text{corr}(\Delta d_t, \Delta pd_t)$, when $i = 1, j = 3$ or $i = 3, j = 1$, and equal to $1 + 2 * \text{corr}(\Delta d_t, \Delta pd_t)^2$, when $i = 2, j = 2$. Significant departure of each empirical moment from its linear counterpart is verified by the [Politis and Romano \(1994\)](#) bootstrap procedure. For the market, ESG, green and high-carbon data, Table 2 shows (in bold) statistical significance at 90%(*) and 95%(**)-level. Dividend growth rate data are positively asymmetric (m_{30}) only for the high-carbon portfolio. Excess kurtosis (m_{40}) is detected for both the market and high-carbon portfolios. Price dividend ratio data are negatively asymmetric (m_{03}) for all portfolios excluding the green one. Excess kurtosis (m_{04}) is detected for all portfolios. Interestingly the excess co-kurtosis (m_{22}) shows significant commonalities between the two series for all market segments excluding the green one. Overall Table 2 confirms the need of a non-linear model to process the data at hand.

needed, data points for recent years are imputed by using the last available observation.

TABLE 2 Third and fourth-order empirical moments

	m_{30}	m_{03}	m_{12}	m_{21}	m_{40}	m_{04}	m_{13}	m_{31}	m_{22}
<i>Market</i>	0.31	-0.58**	0.47**	-0.38	4.01*	4.18**	-3.97**	-3.88*	3.87**
<i>ESG</i>	0.31	-0.43**	0.41**	-0.37	3.87	3.96**	-3.68*	-3.66	3.59*
<i>Green</i>	0.18	0.80	0.02	-0.24	3.78	7.72*	-3.34*	-2.93	2.76
<i>High-carbon</i>	0.47**	-1.84**	-0.13	-0.58**	5.46**	10.87**	-0.71	-3.94	4.52**

Notes: m_{ij} is the empirical standardised moment of order i, j , where i refers to Δd_t , and j to Δpd_t ; * and ** refers to statistical significance at 90% and 95%-level according to the Politis and Romano (1994) stationary bootstrap.

4 Overvaluation in the European market

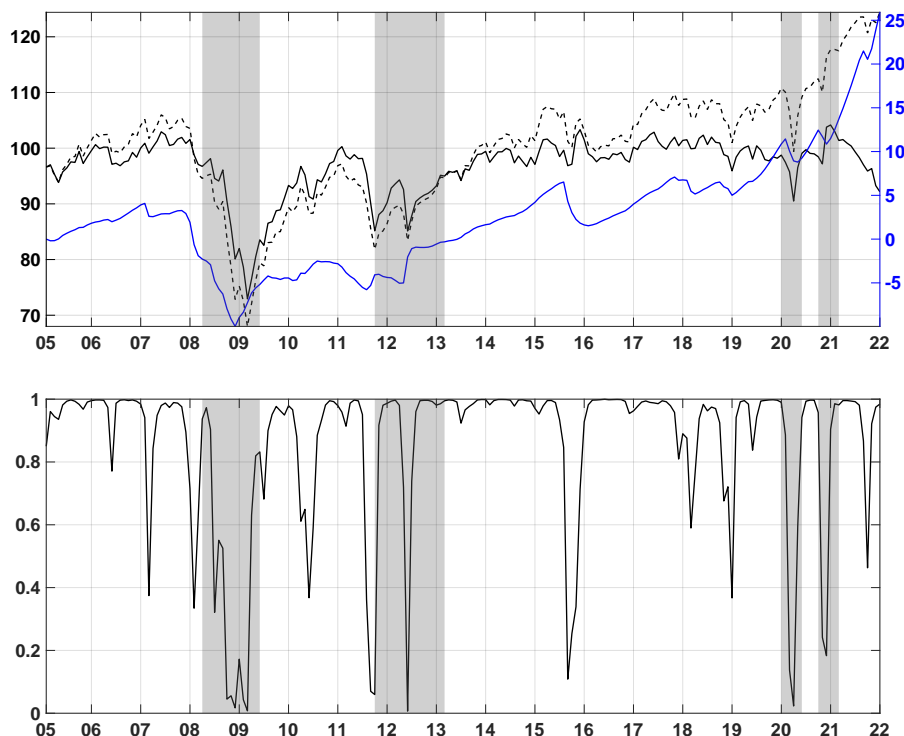
The following subsections present the estimation results for the European market as a whole, as well as for the ESG, green and high-carbon portfolios. Finally, we try to understand how inflationary dynamics have developed in these market niches.

4.1 The market portfolio

In the first part of our analysis, we investigate the role of the non-fundamental component in the European market portfolio. The top panel of Figure 1 shows the price-dividend ratio (dashed black line, left axis) and the posterior mean estimates of the fundamental and non-fundamental (blue) components, i.e. $E(pd_t^f | \mathfrak{S}_T)$ and $E(b_t | \mathfrak{S}_T)$, $t = 1, 2, \dots, T$. The figure also shows periods of EU-27 negative quarterly real GDP growth (shaded areas) to facilitate the interpretation of the components in the context of real economic developments. Indeed, the price components show strong co-movement with economic activity especially during the great recession (2008-2009) and the sovereign debt crisis (2011-2013). The European stock market was slightly overvalued before the great recession and between the end of the sovereign debt crisis and 2020. Since 2016, the model captures a gradual increase in the non-fundamental component, possibly driven by unconventional monetary policy (see Hudepohl et al., 2021).

The bottom panel of Figure 1 shows the smoothed posterior probability of the surviving

FIGURE 1 Top panel: price-dividend ratio (dashed), fundamental and non-fundamental components. Bottom panel: posterior probability of the surviving regime.

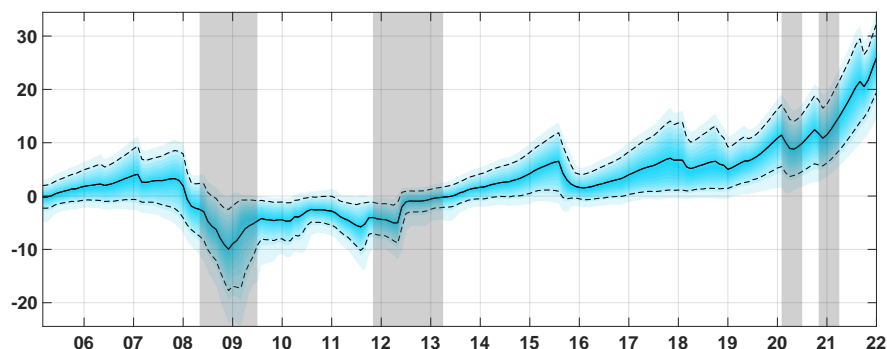


Notes: The sample starts in February 2005 and ends in January 2022. The price-dividend ratio is normalised to 100, and the fundamental is expressed as the relative percentage times the price-dividend ratio. The non-fundamental expressed on the right hand side is percentages over the total price-dividend ratio. Shaded areas describe periods of EU-27 negative quarterly real GDP growth.

regime, $\Pr[S_t = 0 | \mathcal{S}_T]$, $t = 1, 2, \dots, T$. We define a posterior probability lower than one half as a collapse. The collapses of the non-fundamental component are strongly related to economic recessions, which provides validation to the results. Moreover, there are two further episodes in 2007 and 2015, when the non-fundamental component collapses, which correspond to a weak global economic momentum.

Finally, Figure 2 shows the posterior mean (black line) and the estimated uncertainty for the non-fundamental component. The dashed lines denote the 10-th and 90-th percentiles of the posterior distribution at each point in time. The colored area includes the extremes of the density distribution. Periods with significant departures from zeros, at the 80% confidence level, are observed during the double-dip recession and the very last sample period. By 2021, we see a surge in the non-fundamental component from 12 to 25% of the total price. While acknowledging a higher estimation uncertainty in the last periods, this strong increase is

FIGURE 2 Posterior mean, 10-th and 90-th percentile (dashed) for the non-fundamental component.



Notes: The sample starts in February 2005 and ends in January 2022. The non-fundamental component is expressed in terms of percentage over the total price-dividend ratio. Shaded areas describe periods of EU-27 negative quarterly real GDP growth.

likely the result of a combination of factors unfolding their effects. Among them, the pandemic emergency purchase programme, a non-standard monetary policy program by the ECB to fight the pandemic. In addition, many small stock investors entered the market and the EC announced their “rebuild better” legislative and fiscal programmes.

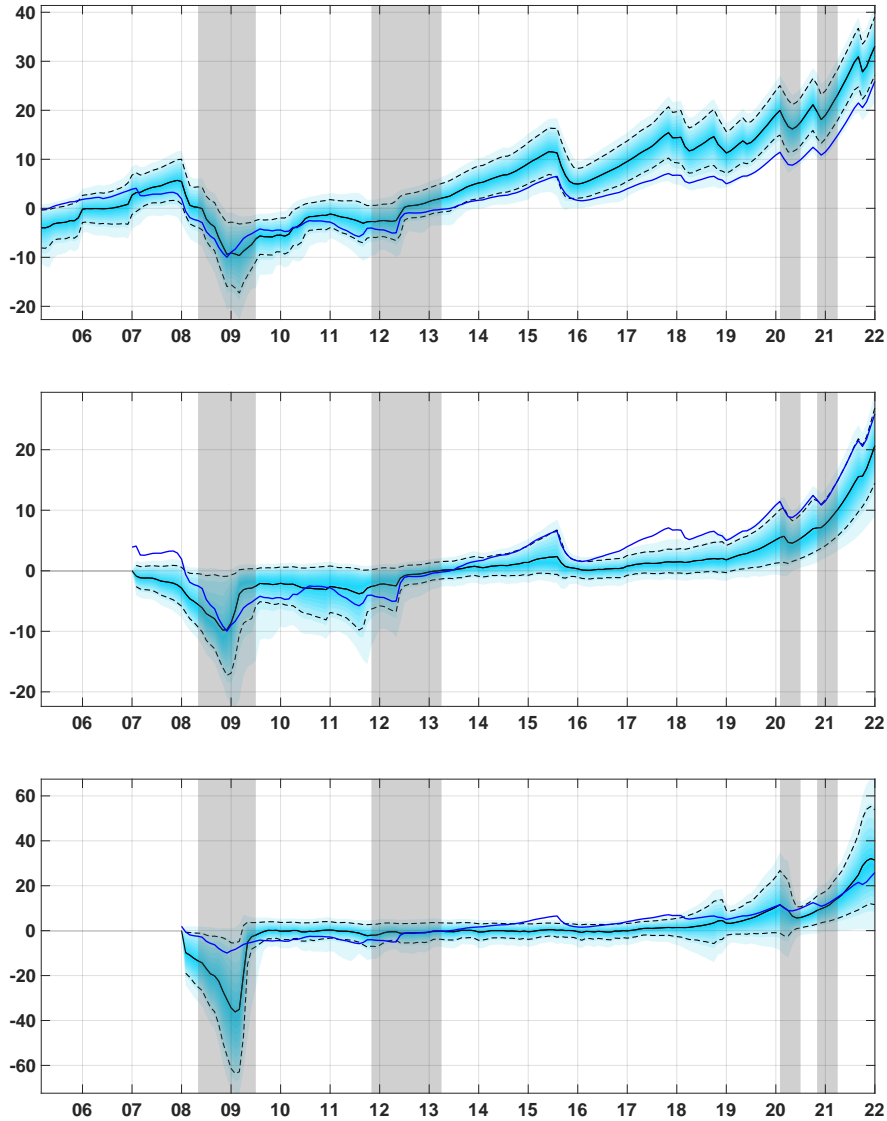
Further estimation results such as the plots exhibiting the prior and posterior distribution of model parameters, the inefficiency factors of the simulated chain, and the posterior mean of the innovations can be found in Appendix C.1. This appendix also includes some robustness checks. In particular, we find that modeling the non-fundamental component as a simple random walk, albeit yielding broadly similar dynamics, results in a comparatively higher dispersion of the estimate.

4.2 ESG, green, and high-carbon portfolios

The abnormal stock market rally since 2021 caused much debate whether the market or only some niches are inflated and whether there is “too much, too quickly of a good thing” with respect to ESG assets.¹¹ We try to answer this question by performing the same analysis as above for relevant market niches, notably ESG, green and high-carbon stocks. We find broadly similar results, but also some important differences.

¹¹20 September 2021: “Central bank group BIS warns of green asset bubble risk” <https://www.reuters.com/business/sustainable-business/global-markets-bis-esg-urgent-2021-09-20/>.

FIGURE 3 Non-fundamental components: posterior mean, 10-th and 90-th percentile (dashed) for the ESG (top panel), Green (mid panel), and High-carbon (bottom panel) portfolios.



Notes: The sample starts in February 2005 (ESG), January 2007 (green) and January 2008 (high-carbon) and ends in January 2022. The blue line is the posterior mean of the non-fundamental component for the market portfolio. The non-fundamental component is expressed in terms of percentage of the total price-dividend ratio. Shaded areas describe periods of EU-27 negative quarterly real GDP growth.

The top panel in Figure illustrates the posterior mean (black line) and the estimated uncertainty for the non-fundamental component of the ESG portfolio, as well as the posterior mean (blue line) of the non-fundamental component of the market portfolio. The non-fundamental components have the same weight in the two portfolios till 2016, when the

non-fundamental component starts to gradually become more important in the ESG portfolio. This could be an indication that investors valued sustainability already back then.

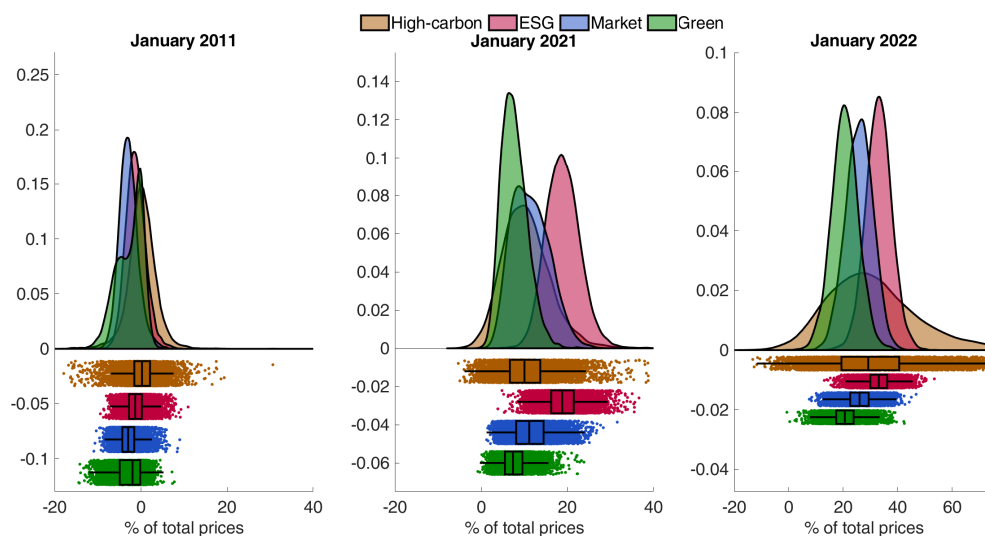
The mid panel displays the non-fundamental component of the green portfolio. Dynamics have been historically less volatile compared to the non-fundamental components for the market or the ESG portfolio. Interestingly, the non-fundamental component of the green portfolio seems having been less affected by unconventional monetary policy measures prior to 2020. By 2020, possibly as a result of further policy announcements, the green non-fundamental component started to increase in line with the market.

Finally, the bottom panel displays the posterior mean and estimated uncertainty for the non-fundamental component of the high-carbon portfolio. Compared to its market counterpart, it is surrounded by a higher estimation uncertainty, due to the higher volatility in the price-dividend ratio in this market segment. Already during the great recession, the estimation uncertainty was huge. Since late 2019, the price-dividend ratio in the high-carbon portfolio started to rise in line with the market. However, also the estimation uncertainty on the size of the non-fundamental component became increasingly large, particularly since 2021. This might reflect uncertainties as regards the actual value of these assets.

4.3 Contrasting non-fundamentals across market segments

The EU Green Deal formulates various incentives for investors to prefer sustainable assets. In a context where exceptional monetary policy measures and fiscal packages were put in place, with an impact on the market as a whole, the question is whether these policies had a stronger impact on sustainability-linked assets. To understand whether some market niches have been significantly more inflated than others, we compare density estimates of various portfolios' non-fundamental components at different points in time. This is of practical relevance for assessing which segments are exposed to sudden collapses in equity prices, but also to investigate whether EU legislation proved successful in shifting investors' preferences. The main results are summarised in Figure 4, while Figure C.6 in the Appendix provides a more detailed picture. The findings suggest that by January 2011, in between the double-dip recession, the non-fundamental components of all portfolios, including the market as a whole, were aligned. In particular, they were all close to zero meaning that

FIGURE 4 Relative inflatedness of non-fundamental components across market segments



Notes: The density estimates refer to the non-fundamental component that is expressed in percentage points of the total price. The boundaries of the boxplot coincide with the second and third quartile.

prices were in line with the fundamental values of the stocks. This is as expected, as indeed by 2011 the sustainability discussion in finance hadn't even started. Even after the stock market rally in the second half of 2020, by January 2021 the non-fundamental component for the entire European stock market only accounted for 12% of total prices. In fact, the non-fundamental component of the green portfolio had about the same weight as the one of the market during the pandemic, which discredits the often made claims of a green bubble. In contrast, by January 2021 the ESG non-fundamental component accounted for almost 20% of the total price. As shown in Figure 4, this difference is highly statistically significant. This is possibly the result of a massive capital inflow into the ESG segment (e.g. see [van der Beck, 2021](#)). Since then, the ESG portfolio has been leading developments but the market did follow. As a result, the overvaluation of the ESG portfolio remains in line with the one of the total market. The non-fundamental component of the ESG portfolio achieved a maximum of 35% in January 2022, which however corresponds to the end of the sample and is therefore associated with higher estimation uncertainty. These findings, coupled with the high uncertainty in the high-carbon portfolio, suggest that a transition towards sustainability may be gaining traction in financial markets.

5 Concluding remarks

In response to the Paris climate agreement in 2015, EU policy institutions announced measures to promote sustainable economic growth. In particular, policies were set in place to ease the transition towards a lower-carbon economy by shifting investor preferences towards sustainable activities. This paper tries to answer the question whether, against a background of unprecedented inflation in European stock prices, a ‘green bubble’ or ‘sustainability bubble’ might be inflating. To investigate to what degree stock prices are deviating from economic fundamentals in various market niches, we utilise a present-value model that allows to estimate the size of the non-fundamental component in stock prices, as well as the likelihood of a ‘burst’.

The model unveils that the entire EU stock market suffers from a substantial deviation from its underlying fundamental values by January 2022, with the non-fundamental component constituting about 25% of total prices. While the green portfolio is in line with the market, the ESG portfolio appears slightly overvalued as compared to the market. These findings suggest that EU regulation starts having some impact on shifting investor preferences towards sustainability objectives as measured by firms’ ESG scores, suggesting a “transition on the move” in financial markets. In support of this argument, markets appear uncertain about the actual value of high-carbon assets, which is reflected in huge uncertainty around the estimates of the fundamental component of the price for this category of assets. All in all, while we find a statistically significant difference between the ESG segment and the market as a whole, the overvaluation in this particular market segment is only slightly larger compared to the market as a whole - at least for the moment. Based on these results, a careful monitoring of the ESG niche is warranted, due to the large inflow of capital into this segment. However, the inflatedness of the entire market seems to be the major concern, also considering that abrupt market adjustments may unfold in response to shocks of various nature.

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A Data construction

We use monthly firm-level data such as the unadjusted price (PU), market value (MV), measure for the number of shares (NOSH, gaps filled with WC05301), and the dividend yield (DY) for 1200 companies that are listed in Europe with all data in Euro.¹² In a first step, we calculate aggregate variables from firm-level data weighting them with the market value. We construct an aggregate price index (PI) for each portfolio j with N being the number of companies in the portfolio.¹³

$$PI_t^j = PI_{t-1}^j \cdot \frac{\sum_{i=1}^N PU_{i,t} \cdot NOSH_{i,t}}{\sum_{i=1}^N PU_{i,t-1} \cdot NOSH_{i,t-1}} \quad \text{with } PI_0^j = 100 \quad (\text{A.11})$$

The aggregate dividend yield for portfolio j is calculated as

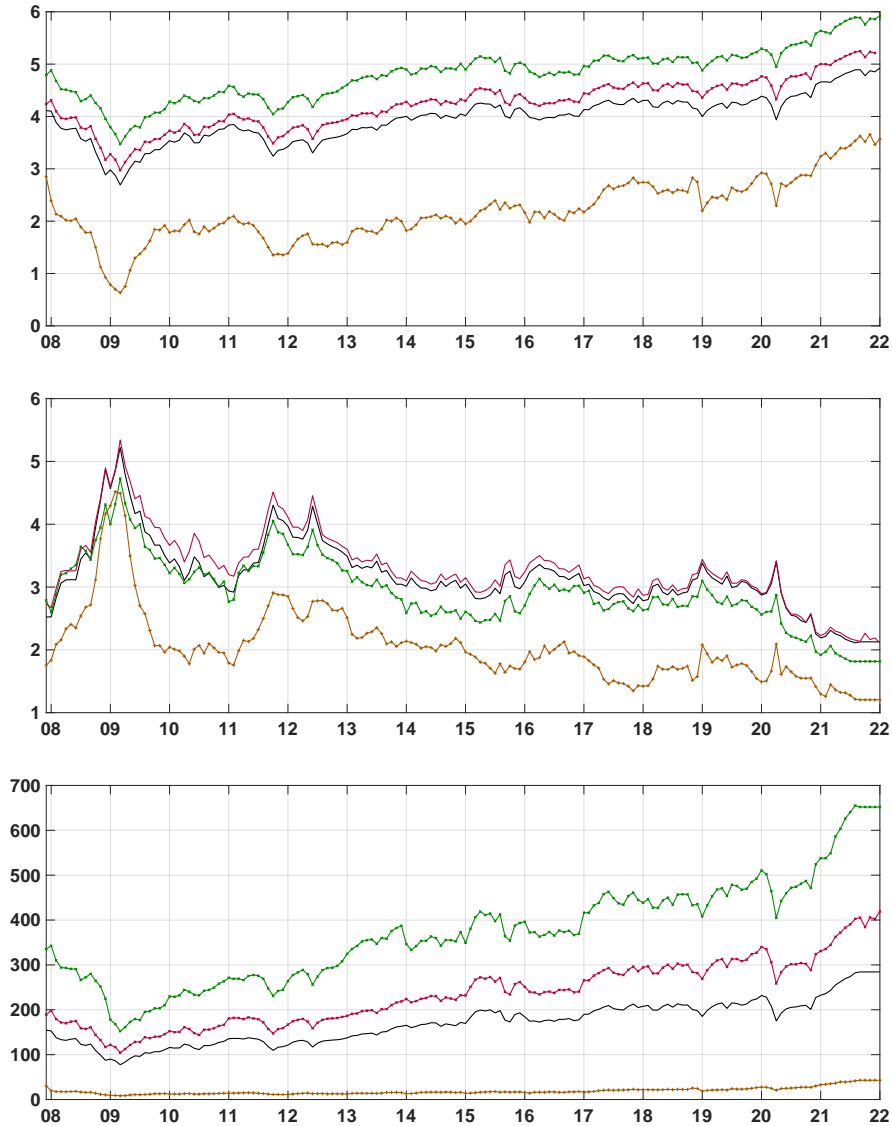
$$DY_t^j = \left(\frac{\sum_{i=1}^N DY_{i,t} \cdot MV_{i,t}}{\sum_{i=1}^N MV_{i,t}} \right) \quad (\text{A.12})$$

The figure below reports raw data for the four markets.

¹²To correct for cases where the company market data should not be denominated in Euro, we apply the DS currency conversion function.

¹³To validate the proposed aggregation procedure in the context of non-availability of DS aggregate market data for the EU, we've applied it to the US constituents listed in the Datastream US market index. The algorithm can recover the aggregate variables though with minor differences.

FIGURE A.1 Raw data for market (black), green (green), high-carbon (brown) and ESG (red). Top panel: price-dividend ratio. Mid panel: dividend yield. Bottom panel: price index.



Note: The charts display the evolution of the price-dividend ratio for the EU market portfolio, the Refinitiv time-varying ESG score (>50) portfolio, as well as the time-varying environmental portfolios (green, high-carbon) as provided by Alessi et al. (2021). Early periods are not displayed due to small firm coverage at the beginning of some series.

B Bayesian estimation

This appendix details the Bayesian approach used for estimating the Markov-switching model reported in the main text. The first section explains how to cast the model in a state-space form, the second section is devoted to the prior elicitation, finally the third

section highlights the MCMC algorithm employed to approximate the posterior distribution of the unobserved states and model parameters.

B.1 State-space format

The model described by equations (8)-(10) admits the general state-space representation:

$$\begin{aligned}\mathbf{y}_t &= \mathbf{H}\xi_t + \mathbf{G}\mathbf{u}_t \\ \xi_t &= \mathbf{a}_t + \mathbf{F}_t\xi_{t-1} + \mathbf{R}_t\mathbf{u}_t\end{aligned}\tag{B.13}$$

where $t = 1, \dots, T$, $\mathbf{y}_t = (pd_t, d_t)'$ is the 2×1 vector of observed variables, $\xi_t = (b_t, pd_t^f, gm_t, gm_{t-1}, g_t, g_{t-1}, \mu_t, e_t^{d*}, e_t^{g*}, e_t^{\mu*}, e_t^{b*})'$ is the 11×1 state vector, and $\mathbf{u}_t = (e_t^d, e_t^g, e_t^\mu, e_t^b)'$ is the 4×1 vector of shocks. The possibly time-varying vectors and matrices, determined by the model parameters and by the discrete latent variables S_t , are equal to:

$$\mathbf{H} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix};$$

\mathbf{G} is a 2×4 matrix of zeros;

$$\mathbf{a}_t = \left(0 \quad \frac{\ln(1+\exp(\bar{p}d)) - \rho_{S_t}\bar{p}d - \alpha_{S_t}^\mu + \alpha_{S_t}^g}{1 - \rho_{S_t}} \quad \alpha_{S_t}^g \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \right)'$$

$$\mathbf{F}_t = \begin{pmatrix} 1 - \rho_{S_t} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\phi_{S_t}^g}{1 - \rho_{S_t} \phi_{S_t}^g} & 0 & \frac{\phi_{S_t}^\mu}{1 - \rho_{S_t} \phi_{S_t}^\mu} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_{S_t}^g & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \phi_{S_t}^\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

and

$$\mathbf{R}_t = \begin{pmatrix} V^b(S_t) & 0 & 0 & 0 \\ 0 & \frac{V^g(S_t)}{1 - \rho_{S_t} \phi_{S_t}^g} & 0 & 0 \\ 0 & 0 & \frac{V^\mu(S_t)}{1 - \rho_{S_t} \phi_{S_t}^\mu} & 0 \\ 0 & V^g(S_t) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & V^\mu(S_t) & 0 \\ V^d(S_t) & 0 & 0 & 0 \\ 0 & V^g(S_t) & 0 & 0 \\ 0 & 0 & V^\mu(S_t) & 0 \\ 0 & 0 & 0 & V^b(S_t) \end{pmatrix}.$$

where $V^x(S_t) = S_t \sqrt{V^x} + (1 - S_t) \sqrt{V^x \delta^x}$, $x = d, b, g, \mu$.

B.2 Prior distributions

Let θ denote the full set of model parameters, i.e. $\theta = (\rho_j, \alpha_j^x, \phi_j^x, V^y, \delta^y)$, $j = 1, 2$, $x = g, \mu$, $y = d, b, g, \mu$, and $\pi = (\pi_{00}, \pi_{01})$ the vector of transition probabilities to be estimated (the remaining probabilities satisfy $\pi_{10} = 1 - \pi_{00}$, and $\pi_{11} = 1 - \pi_{01}$). We assume:

A0. All parameters in θ are a priori independent, and θ is independent from π .

Furthermore we elicitate the following densities:

$$\rho_j \sim N(m_\rho, s_\rho)1_A, \quad j = 0, 1.$$

$$\alpha_j^x \sim N(m_{\alpha^x}, s_{\alpha^x}), \quad j = 0, 1, x = g, \mu.$$

$$\phi_j^x \sim N(m_{\phi^x}, s_{\phi^x}), \quad j = 0, 1, x = g, \mu.$$

$$V^y \sim IG(m_{V^x}, s_{V^x}), \quad y = d, g, \mu, b.$$

$$\delta^y \sim B(m_{\delta^x}, s_{\delta^x})1_A, \quad y = d, g, \mu, b.$$

$$\pi \sim \text{Dirichlet}(\mathbf{a}_1, \mathbf{a}_2).$$

where $N(m, s)1_A$ is the Gaussian distribution with mean m and variance s , defined over $A \subset \mathfrak{R}$, $IG(m, s)$ is the inverted gamma distribution with mean equal to $m/(s - 2)$ and variance $2m^2/((s - 2)^2(s - 4))$ (see [Bauwens et al., 1999](#)), and $B(m, s)1_A$ is the (shifted) beta distribution with support $A \subset \mathfrak{R}$. Since the number of states of the Markov process is two, assuming a Dirichlet distribution for π is equivalent to elicitate independent Beta distributions for p_{00} and p_{01} . Table [B.3](#) reports the mean, standard deviation, and the extremes of the support A , specified in the empirical exercise.

The bounds on (ρ_0, ρ_1) are imposed to identify collapsing and surviving regimes, those on (ϕ_0^x, ϕ_1^x) are imposed to enforce stationarity, and the bounds on δ^y impose a larger variance for the collapsing regime. The priors on π_{00} and π_{01} assume highly persistent states. The idea is to have a low volatility for the \mathbf{S} sequence. The parameter \bar{pd} that appears in [\(9\)](#) is set to the time series average of pd_t .

TABLE B.3 Prior distributions

	type	mean	sd	lb	ub
ρ_0	N	0.87	0.01	0	0.95
ρ_1	N	1.15	0.01	1.05	1.25
α_j^x	N	0	0.05	$-\infty$	∞
ϕ_j^x	N	0.8	0.05	0	.99
V^y	IG	0.001	0.001	0	∞
δ^y	B	34	17.64	1	100
π_{00}	B	0.96	0.04	0	1
π_{01}	B	0.04	0.04	0	1

Notes: $j = 0, 1$, $x = g, \mu$, $y = d, g, \mu, b$, sd denotes standard deviation, lb and ub denote the extremes of the support.

B.3 Posterior distributions

For any variable \mathbf{w}_t let \mathbf{w} denote the vector $\mathbf{w} = (\mathbf{w}_1, \dots, \mathbf{w}_T)$. In order to derive the MCMC scheme which delivers draws from the posterior distributions $p(\theta, \pi, \mathbf{S}, \xi | \mathbf{y})$, we rely on the following assumptions:

- A1. Shocks are independent and Gaussian: $\mathbf{u}_t \stackrel{\text{iid}}{\sim} N(\mathbf{0}, \mathbf{I})$.
- A2. S_t is a Markov process with transition probabilities $\pi_{ij} \equiv \Pr(S_t = i | S_{t-1} = j)$, $i, j = 1, 2$.
- A3. The system matrices \mathbf{a}_t , \mathbf{F}_t , and \mathbf{R}_t are known function of the model parameters θ and of the contemporaneous variable S_t only.
- A4. Given S_t , the matrices \mathbf{a}_t , \mathbf{F}_t , and \mathbf{R}_t , do not depend on the transition probabilities π .

As underlined in A1 we focus on conditionally Gaussian state-space models. Assumption A2 restricts the dynamics of the discrete latent variables to Markov processes. Assumptions A3 and A4 are standard hypothesis that help simplifying the MCMC simulations. Furthermore we allow the matrix \mathbf{F}_t to have real eigenvalues greater than one. This circumstance is tackled by the diffuse Kalman filter initialisation put forward by [Koopman \(1997\)](#).

Samples from the joint posterior distribution $p(\theta, \pi, \mathbf{S}, \xi | \mathbf{y})$ are obtained using the factorization:

$$p(\theta, \pi, \mathbf{S}, \xi | \mathbf{y}) = p(\xi | \theta, \pi, \mathbf{S}, \mathbf{y}) p(\theta, \pi, \mathbf{S} | \mathbf{y})$$

Posterior samples of the state vector ξ are drawn off-line using the simulation smoother proposed by [Durbin and Koopman \(2002\)](#). Samples from $p(\theta, \pi, \mathbf{S} | \mathbf{y})$ are obtained with the following Gibbs scheme:

$$p(\theta | \mathbf{S}, \pi, \mathbf{y}), \Pr(\mathbf{S} | \theta, \pi, \mathbf{y}), p(\pi | \theta, \mathbf{S}, \mathbf{y})$$

Assumptions A0 and A4 imply that the first full conditional verifies:

$$p(\theta | \mathbf{S}, \pi, \mathbf{y}) \propto p(\mathbf{y} | \mathbf{S}, \theta) p(\theta)$$

The conditionally Gaussian hypothesis A1 makes possible the evaluation of the augmented likelihood $p(\mathbf{y} | \mathbf{S}, \theta)$ by Kalman filtering (see [Kalman, 1960](#)). For non-stationary state variables, diffuse initial conditions are handled as in [Koopman \(1997\)](#). For stationary state variables say ξ_t^* , the recursions are initialised using the unconditional mean and covariance matrix $E(\xi_1^* | \theta, \mathbf{S})$ and $V(\xi_1^* | \theta, \mathbf{S})$. The unconditional covariance matrix of the stationary elements of the state vector is calculated as in [Kitagawa \(1977\)](#). Draws of θ are obtained one parameter at-a-time from the full conditional distribution $p(\theta_i | \theta_{-i}, \mathbf{S}, \mathbf{y})$ using the stepping out slice sampler proposed by [Neal \(2003\)](#).

The \mathbf{S} -sequence is drawn with the multi-move adaptive MH sampler given in [Fiorentini et al. \(2014\)](#) which reduces chain autocorrelation when the discrete latent variables are conditionally dependent. It samples S_t in blocks: given a block length h , the multi-move sampler draws from $\Pr(\mathbf{S}_t, \mathbf{S}_{t+1}, \dots, \mathbf{S}_{t+h-1} | \mathbf{S}_{-t, \dots, t+h-1}, \theta, \pi, \mathbf{y})$. This scheme yields the full \mathbf{S} sequence marginally to ξ in $O(T)$ operations.

It remains to show the sampler of π given \mathbf{S} , θ , and \mathbf{y} . Assumptions A0 and A4 imply that

given \mathbf{S} , π does not depend on θ and \mathbf{y} : $p(\pi|\theta, \mathbf{S}, \mathbf{y}) \propto \Pr(\mathbf{S}|\pi)p(\pi)$. By factorizing we get:

$$\begin{aligned}
p(\pi_{00}, \pi_{01}|\mathbf{S}) &\propto \Pr(\mathbf{S}|\pi_{00}, \pi_{01}) \prod_{k=0}^1 p(\pi_{0k}) \\
&\propto \Pr(S_1|\pi_{00}, \pi_{01}) \prod_{t=2}^T \Pr(S_t|S_{t-1}, \pi_1, \pi_2) \prod_{k=0}^1 p(\pi_{0k}) \\
&\propto \Pr(S_1|\pi_{00}, \pi_{01}) \prod_{k=0}^1 \prod_{t \in I_k} \Pr(S_t|S_{t-1} = k, \pi_{0k}) p(\pi_{0k})
\end{aligned}$$

where $I_k = \{t \geq 2 : S_{t-1} = k\}$. The term $\prod_{k=0}^1 \prod_{t \in I_k} \Pr(S_t|S_{t-1} = k, \pi_{0k}) p(\pi_{0k})$ is proportional to the product of independent Dirichlet distributions. Under the assumption $\pi \sim \text{Dirichlet}(a_1, a_2)$, the full conditional $p(\pi|\mathbf{S})$ is a Dirichlet distribution with hyperparameters (α_1^*, α_2^*) such that:

$$\alpha_j^* = \alpha_j + \sum_{t=1}^T \mathbf{1}_{(S_t=j)}, \quad j = 1, 2$$

where $\mathbf{1}_{(\cdot)}$ is the indicator function.

To remove dependence on the initial condition S_1 , the term $\prod_{k=0}^1 \prod_{t \in I_k} \Pr(S_t|S_{t-1} = k, \pi_{0k}) p(\pi_{0k})$ is taken as proposal in a MH step with acceptance probability given by $\min\{1, \Pr(S_1|\pi^*) / \Pr(S_1|\pi)\}$, where π^* is the candidate vector and π is the previously sampled value.

In all empirical exercises reported in the main text we record ten thousand draws after discarding the five hundred from the scheme outlined above. Estimates of the unobserved components ξ_t , $t = 1, \dots, T$ are obtained as the average $(1/G) \sum_{g=1}^G \xi_t^g$. These averages are estimates of the conditional expectation $E(\xi_t|\mathbf{y})$. As such they take into account parameter uncertainty.

To illustrate the uncertainty incorporated in the estimate, we apply fan charts (blue fan) to display the non-fundamental component. The differing layers of the fan represent the underlying percentiles. To improve the interpretability of these charts, the 10 and 90 percent confidence intervals are shown in the form of dashed black lines. Further, the uncertainty at a specific point in time is illustrated by using so called rain-cloud plots. Based on the

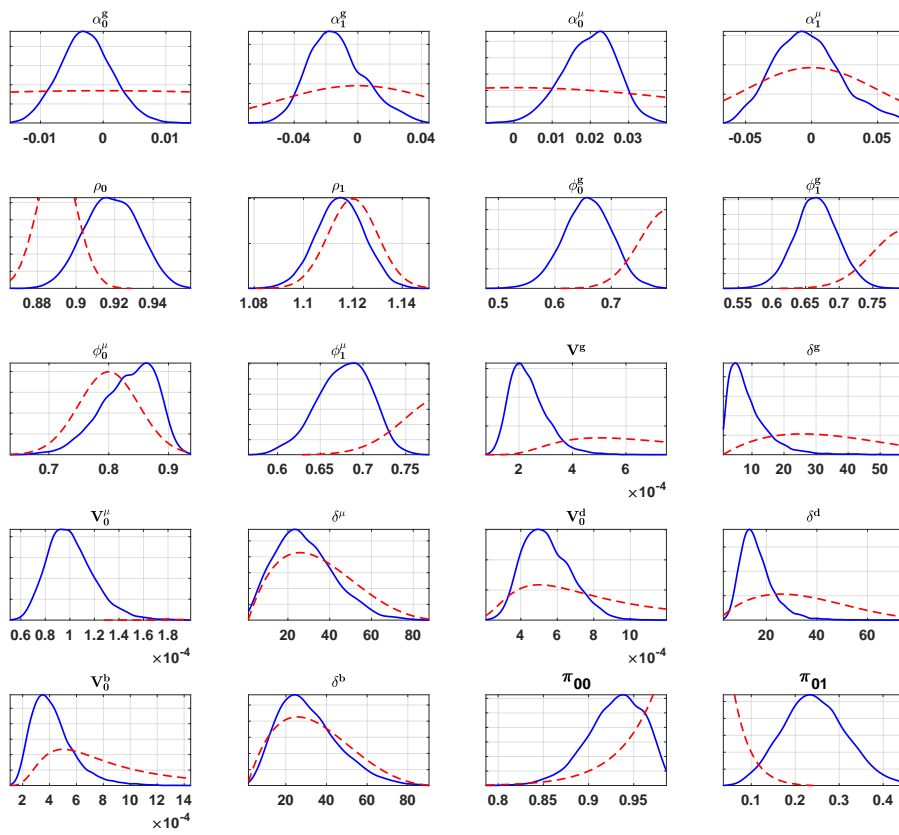
estimated values at one point in time, the kernel density is calculated for each portfolio. Below, the estimates from the draws are plotted in one band with a box plot overlaying the observations. The y-data doesn't have a meaning to these plots. The box defines the median as well as the lower and upper quartiles, while the whiskers define the minimum and maximum which are the lower/upper quartiles minus/plus the interquartile range.

C Additional results

C.1 Detailed estimation results for the European composite portfolio

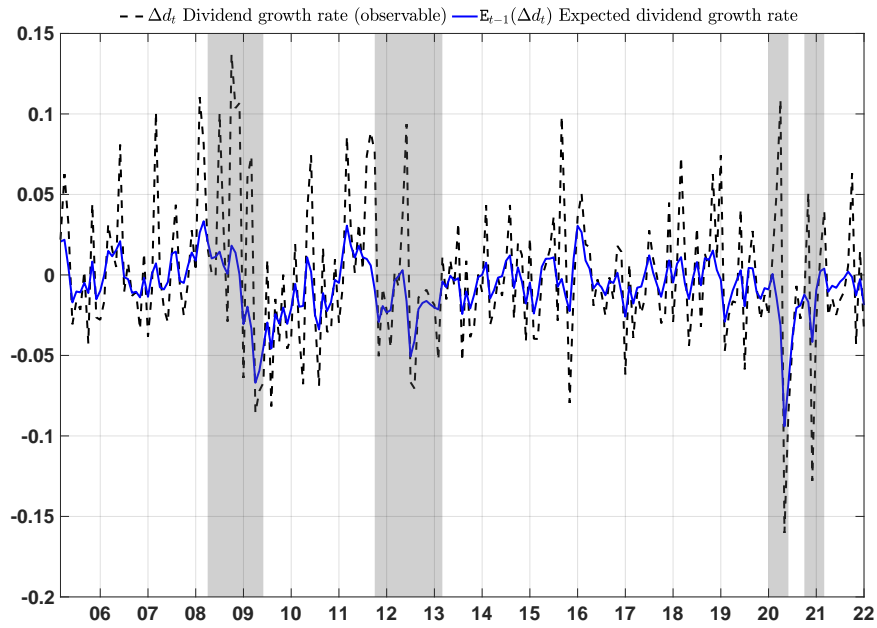
The figure below shows kernel estimate of posterior distribution of model parameters together with the relative prior distribution (red-dashed).

FIGURE C.2 Prior and posterior distributions, European market portfolio



Notes: The graph illustrates prior (red) and posterior (blue) distributions for all estimated parameters for the European market portfolio. The sample span ranges from February 2005 to January 2022.

FIGURE C.3 Dividends and expected dividends, European market portfolio



Notes: The graph reports the dividend growth rate data (dashed) and the expected dividend growth rates (blue). Shaded areas describe periods of EU-27 negative quarterly real GDP growth. The sample starts in February 2005 and ends in January 2022.

Table C.1 reports the lag(k)-autocorrelation ($k = 1, 10, 100$) and the inefficiency factor (IF) of the draws generated by our MCMC scheme used to estimate features of the posterior distribution of model parameters. The inefficiency factor is defined as follows:

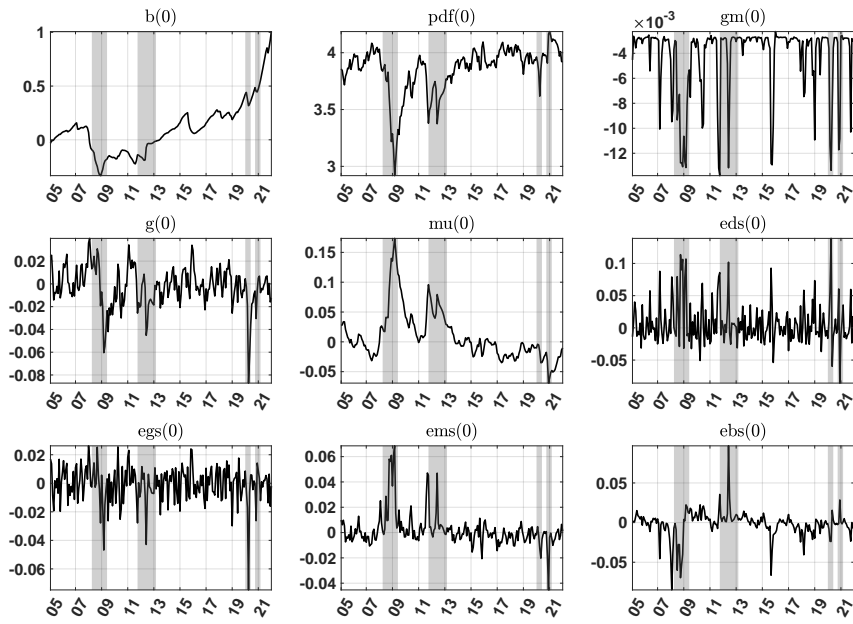
$$IF = 1 + 2 \sum_{k=1}^L \omega_k \rho_k \quad (\text{C.14})$$

with ρ_k being the lag-k sample autocorrelation of a given parameter, ω_k the Parzen-weights, and $L=500$ the maximum lag length.

TABLE C.4 Autocorrelations and inefficiency factors of posterior draws.

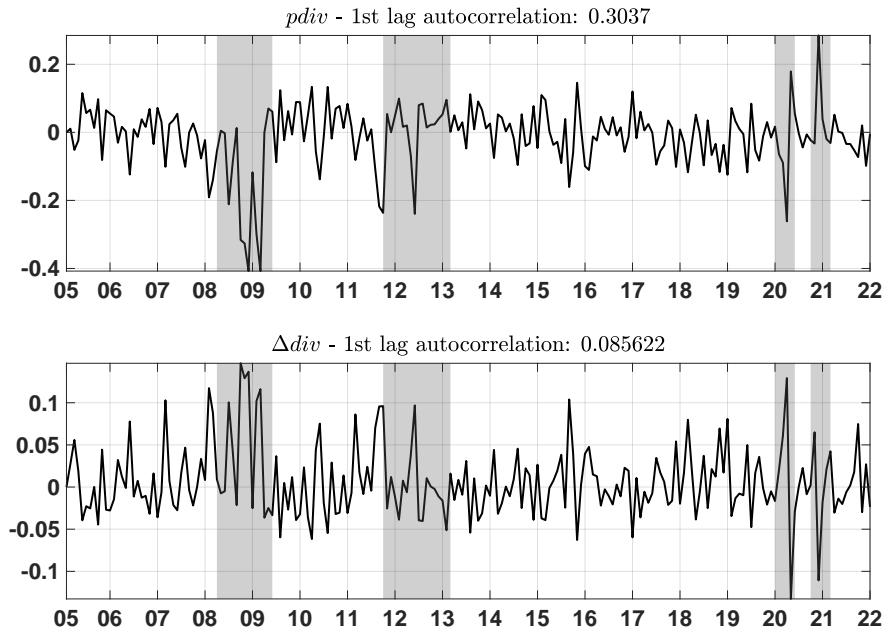
	1	10	100	IF
α_0^g	0.853	0.323	0.003	16.081
α_1^g	0.981	0.846	0.204	96.322
α_0^μ	0.934	0.576	0.102	62.315
α_1^μ	0.987	0.886	0.277	117.185
ρ_0	0.786	0.371	0.098	45.122
ρ_1	0.312	0.019	0.009	2.639
ϕ_0^g	0.529	0.028	0.015	7.018
ϕ_1^g	0.495	0.053	-0.004	6.452
ϕ_0^μ	0.919	0.586	0.150	69.047
ϕ_1^μ	0.770	0.325	0.084	35.570
V^g	0.711	0.250	0.071	32.428
δ^g	0.642	0.199	0.073	30.342
V_0^μ	0.485	0.023	0.003	5.998
δ^μ	0.623	0.328	0.098	43.011
V_0^d	0.739	0.367	0.086	41.630
δ^d	0.431	0.050	-0.007	5.368
V_0^b	0.445	0.027	0.007	2.470
δ^b	0.361	0.021	0.012	5.332
b	0.123	0.045	0.006	6.043

FIGURE C.4 Unobservable states



Notes: The graphs illustrate the estimated unobservable states. Shaded areas describe periods of EU-27 negative quarterly real GDP growth. The sample starts in February 2005 and ends in January 2022.

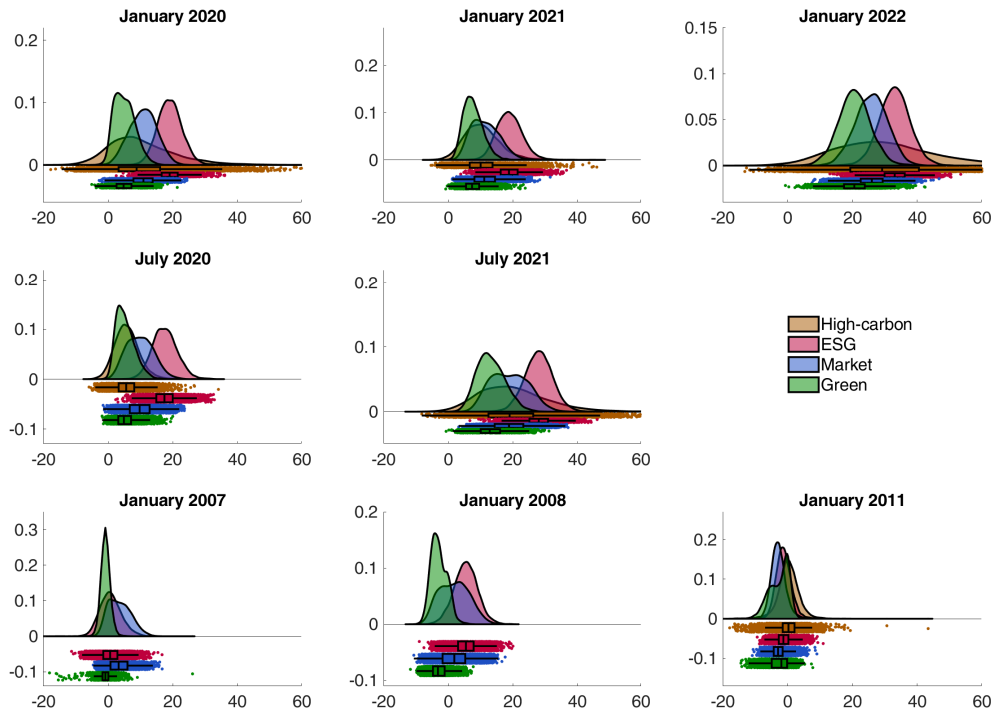
FIGURE C.5 Innovations



Notes: The graphs illustrate the model innovations $y_t - E(y_t | \mathfrak{F}_{t-1})$. Shaded areas describe periods of EU-27 negative quarterly real GDP growth. The sample starts in February 2005 and ends in January 2022.

C.2 Contrasting non-fundamental components

FIGURE C.6 Contrasting non-fundamental components across market segments



Notes: The non-fundamental component is expressed in percentage of the total price. The boundaries of the boxplot describe the second and third quartile.

C.3 Comparison with a naive model

To assess the robustness of our baseline results we compare them with that of a model without Markov switching mechanism where the non-fundamental component evolves as a random walk. This model is described by the observational equations:

$$\begin{aligned}\Delta div_t &= g_{t-1} + e^d \\ pd_t &= pdf_t + b_t\end{aligned}$$

The dynamics of the latent variables are now given by:

$$\begin{aligned}pd_t^f &= (\kappa + \alpha^g - \alpha^\mu)/(1 - \rho) - (\mu_t - \alpha^\mu)/(1 - \rho\phi^\mu) + (g_t - \alpha^g)/(1 - \rho\phi^g) \\ b_t &= b_{t-1} + \epsilon_b^* \\ g_t &= \alpha^g + \phi^g(g_{t-1} - \alpha^g) + e^g \\ \mu_t &= \alpha^\mu + \phi^\mu(\mu_{t-1} - \alpha^\mu) + e^\mu\end{aligned}$$

The shock $(\epsilon^d, \epsilon^g, \epsilon^\mu, \epsilon^b)$ are uncorrelated white noises with variances (V^d, V^g, V^μ, V^b) . The parameters $(\kappa, \rho, \alpha_\mu)$ are calibrated on the basis of the log gross return time series which is computed by eq (1). To the rest of model parameters a flat prior was assigned. Figure C.7 illustrates the comparison between the unobservable states for the market portfolio. The results suggest that the Markov-switching model provides a more conservative estimate of the bubble, or conversely, a better fit of varying fundamentals to the observable data. As indicated in Figure C.8 also the uncertainty around the estimate appears to be smaller in the Markov-switching model as compared to a model where the non-fundamental is modelled as a random walk. The findings support the application of a Markov-switching model in this context.

FIGURE C.7 Unobservable states European market portfolio, Markov-switching bubble process (black), random walk bubble process (blue), data (dashed).

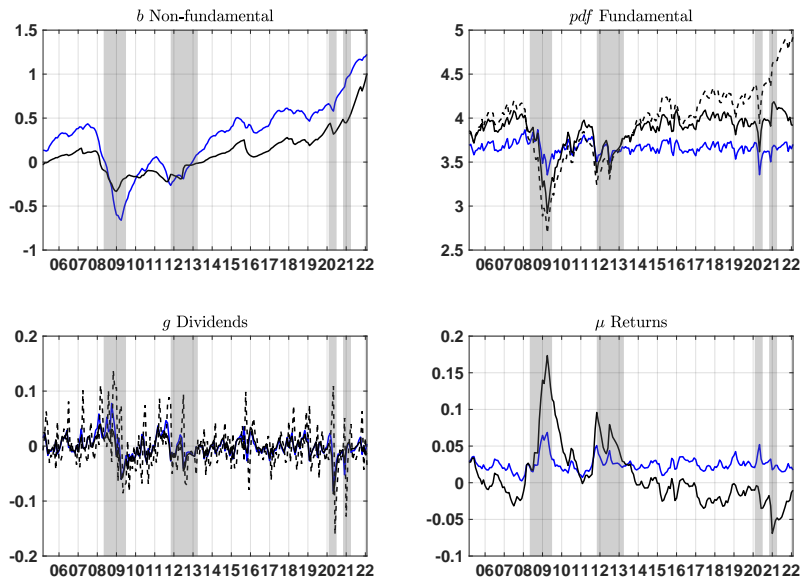
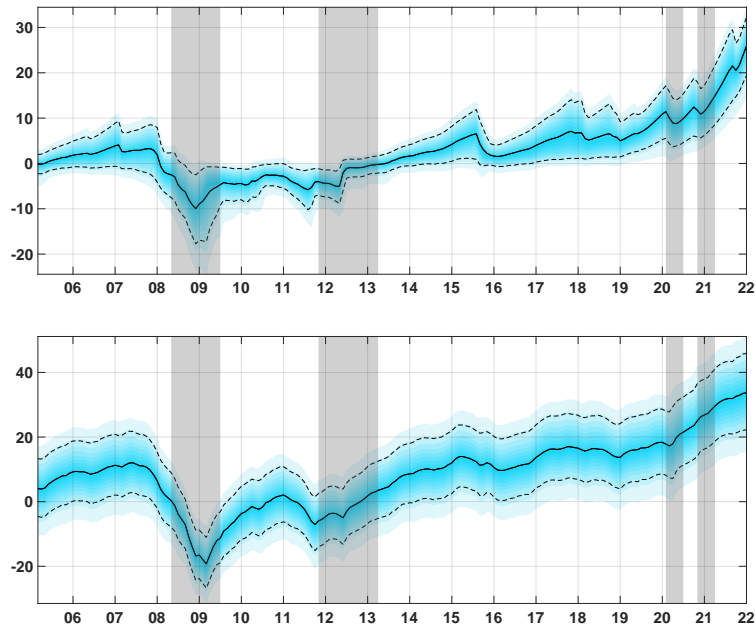


FIGURE C.8 Posterior mean, 10-th and 90-th percentile (dashed) for the non-fundamental component in model with Markov-switching bubble process (top panel) and in model with a random walk bubble process (bottom panel).



Notes: The non-fundamental component is expressed in terms of percentage points of the total price-dividend ratio.



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