Benford testing procedure validation

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Statistics and Information Technologies for Anti-Fraud, Security and Trade

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Objective: study and assess the properties of the Benford testing procedures described in

"Newcomb–Benford law and the detection of frauds in international trade" *Cerioli, A., Barabesi, L., Cerasa, A., Menegatti, M., & Perrotta, D.* (2019). *Proceedings of the National Academy of Sciences, 116(1), 106-115.*

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in terms of ***size***, ***power*** and ***rate of false signals*** on a real environment.

In the article, the proposed procedure is shown to work properly in simulated markets.

However, no data were available for a huge validation experiment.

Description of the proposed procedure:

Trader X:
N transactions
on
M distinct products

- N_1 *transactions on Product 1*
- N_2 *transactions on Product 2*
- …
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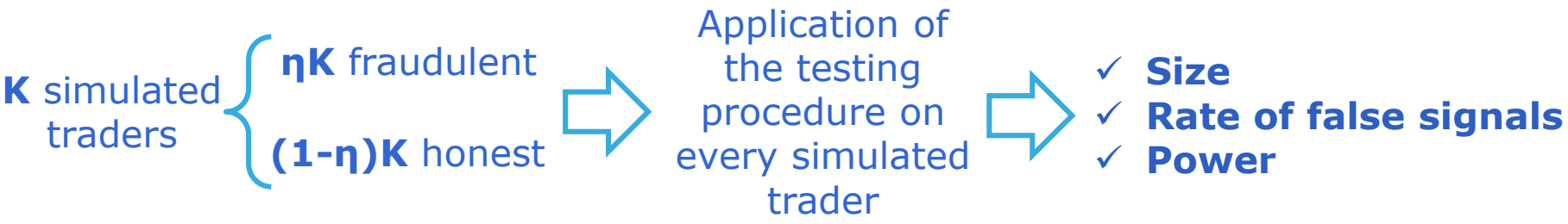
Empirical distribution of the χ^2 on the first digit for **Trader X**

Exact p-value

Simulated markets were used to assess the properties of the procedure:



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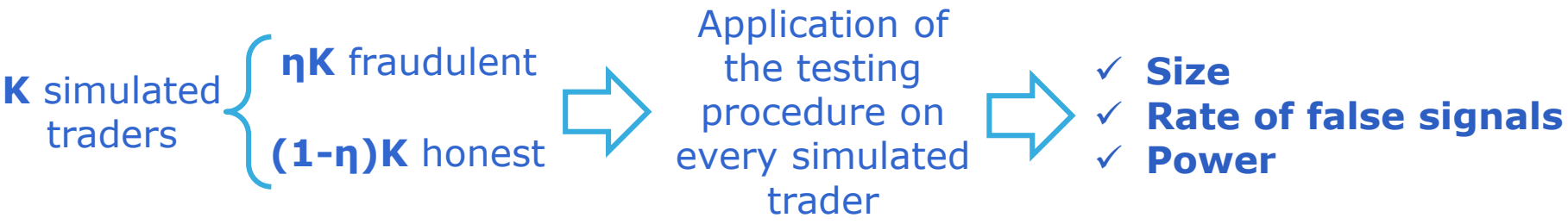

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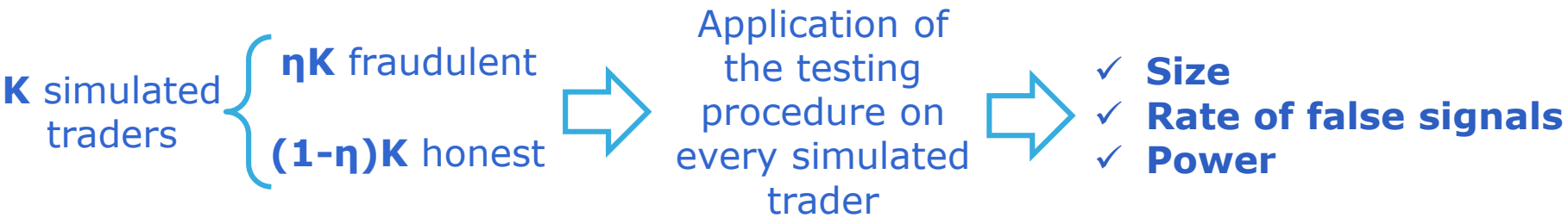

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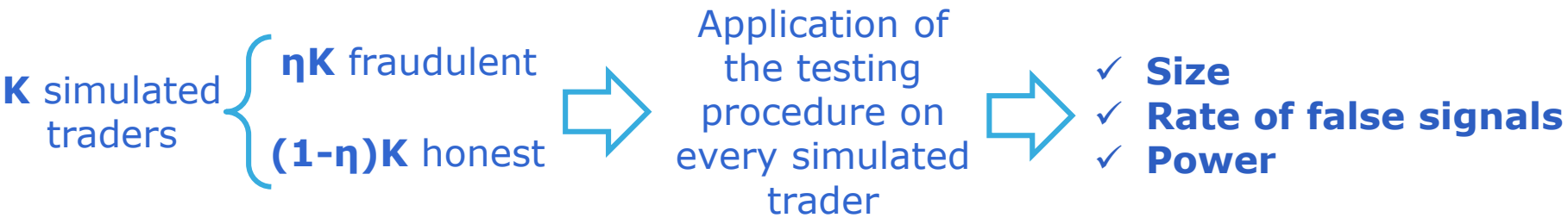

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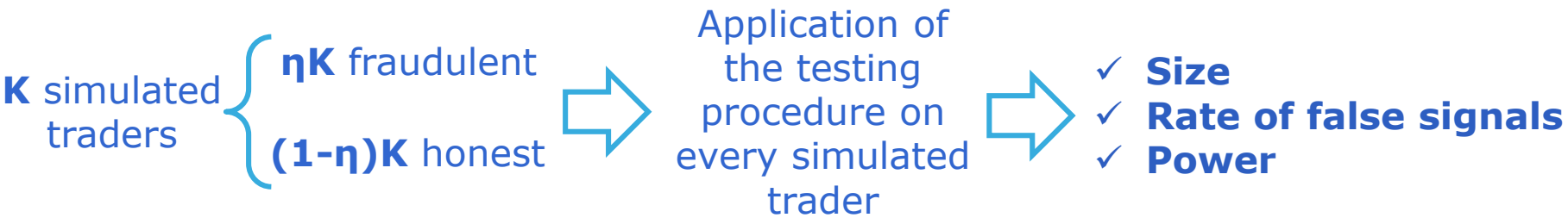

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- ✓ Different fraud strategies and fraud intensities *(defined as the percentage of manipulated transactions)*;
- ✓ Different fractions of fraudulent traders **η**.

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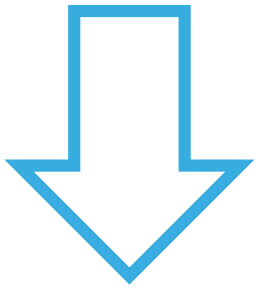

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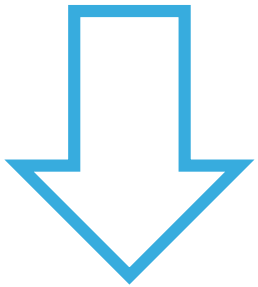

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49,024 regular

6,686 under-evaluation

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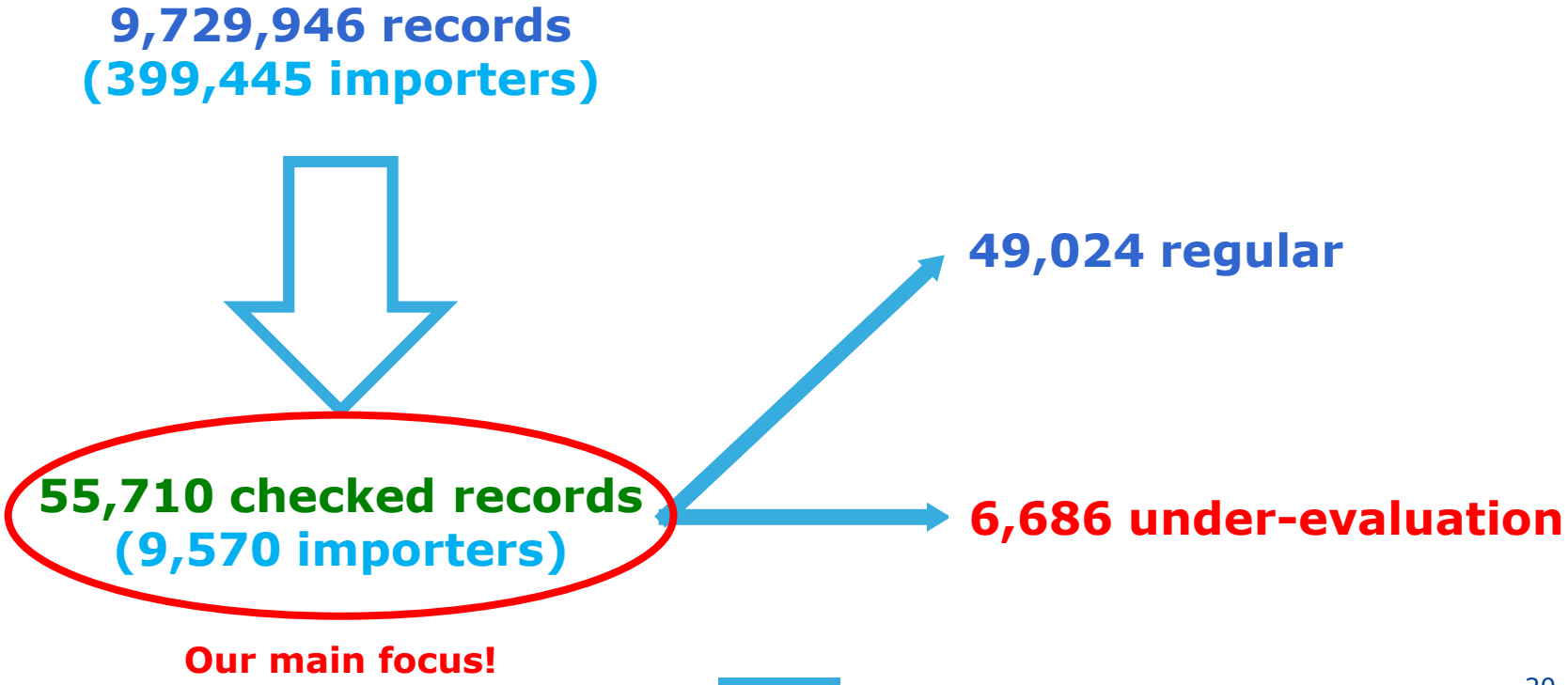

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It is difficult to give a reliable definition of an honest trader

Transactions considered	all	9,729,946
Checked companies considered	traders with at least 50 transactions & at least 1 transaction checked	3,856
Definition of honest trader	no fraud detected	3,337
Definition of fraudulent trader	at least 1 fraud detected	519

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Trader A

100 transactions

99 not-checked

1 checked

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Trader A	**Trader B**
100 transactions	100 transactions
99 not-checked	35 not-checked
1 checked	65 checked
Regular	*All regular*

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Trader A	**Trader B**	**Trader C**
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Results will then consider different definitions of the honest traders set.

Filter 1

Improve the reliability of honest traders definition by considering only the traders with a significant fraction (40%) of transactions checked.

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Rate of False Signals: 18.18%
Power: 75.00%

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Size: 8.00%
Rate of False Signals: 18.18%
Power: 75.00%

Nice results but too few traders…

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Focus only on the 55,710 checked records. In this way, it is possible to exactly discriminate honest and fraudulent traders, since all transactions have been checked.

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Transactions considered	Only checked	55,710
Checked companies considered	traders with at least 50 transactions	143
Definition of honest trader	no fraud detected	82
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Size: 7.32%
Rate of False Signals: 31.58%
Power: 21.31%

Good results in general.
Low power, but also low fraud intensity (median: 12.5%)
Still high size and high rate of false signals…
Further investigations are required…

Analyzing in details the statistical values declared by the 143 traders, we observed that:

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Therefore we decided to further filter the traders…

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Transactions considered	Only checked	55,710
Checked companies considered	traders with at least 50 transactions & Declared Statistical Value Range>3 & Percentage of Repeated Purchases<20%	102
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Rate of False Signals: 25.00%
Power: 15.38%

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Size: 3.17%
Rate of False Signals: 25.00%
Power: 15.38%

Size very close to the nominal level (1%)
Improved rate of false signals
Lower power, but also lower fraud intensity (median: 3.9%)

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- Power seems poor but is directly proportional to the intensity of fraud, which is usually low.
- When a suitable filter is applied, the size of the testing procedure is close to its nominal level.
- An heuristic filter (i.e. filter 3) guarantees the best results in this sense.
- A refined simulation scheme taking into account similarities and dependences of traders behavior will be object of future studies